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This project, funded by AFOSR, ARO, NASA and ONR, was run by the writer with Profs. Brian E. Launder, University of Manchester, England, and John L. Lumley, Cornell University. Statistical data on turbulent flows, from lab. experiments and simulations, were circulated to modelers throughout the world: this is the first large-scale project of its kind to use simulation data. The modelers returned their predictions to Stanford, for distribution to all modelers and to additional participants ('experimenters'); over 100 in all. The object was to obtain a consensus on the capabilities of present-day turbulence models, and identify which types most deserve future support. This has not been completely achieved, mainly because not enough modelers could produce results for enough test cases within the duration of the project. However, a clear picture of the capabilities of various modeling groups has appeared, and the interaction has been helpful to the modelers. The results support the view that Reynolds-stress transport models are the most accurate.

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"Collaborative Testing of Turbulence Models"

1 February 1990 – 30 April 1992

P. Bradshaw, Principal Investigator

Mechanical Engineering Department
Stanford University Stanford, California 94305-3030

December 1992

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SUMMARY

This project, supported by AFOSR, Army Research Office, NASA and ONR, was administered by the writer with Prof. Brian E. Launder, University of Manchester, England and Prof. John L. Lumley, Cornell University. Statistical data on turbulent flows, from lab. experiments and simulations, were circulated to turbulence modelers all over the world. This is the first large-scale project of its kind to use the results of simulations (numerically-exact solutions of the three-dimensional, time-dependent Navier-Stokes equations) and for this reason alone is a landmark in the testing of turbulence models. The modelers compared their "predictions" with the data and returned the results to Stanford, for distribution to all modelers and to additional participants ("experimenters"); over 100 participants in all. The object was to obtain a consensus on the capabilities of present-day turbulence models, and to identify the types of model which most deserved support for future development. This has not been achieved, mainly because not enough modelers could produce results for enough test cases within the duration of the project (our modest request was, roughly, 25 test cases in two years). However a clear picture of the capabilities of various modeling groups has appeared, and the interaction has also clarified the outlook of the modelers themselves. The results support the proposition that Reynolds-stress transport closures (second-moment closures) are more accurate/adaptable, but no account has been taken of their greater cost per calculation.

1. INTRODUCTION

After consideration of a 1992 conference at Stanford, as a linearly-extrapolated successor to the 1968 and 1980-81 meetings (Refs. 1 and 2), the first formal proposal envisaged a 4-year "mail order" effort, hopefully long enough for significant improvements to be made in the models. This was subsequently cut, at the funding agencies' request, to a nominal 18 months, with the object of finding "where we are at in turbulence modeling" (without allowing time for improvements) and then extended to just over 2 years. Although a great deal of useful information has been obtained, it has become sadly clear that very few turbulence modeling groups are both able and willing to compute test cases, covering a wide range of turbulent flows, within a reasonable period of time. Unsurprisingly, some

time was taken in improvement of models and codes, but this was only part of the reason for the delay.

In brief, the quality of the results obtained seems to be much more closely correlated with the competence of the modeler/modeling group, the personnel available to do the actual ruunning of test cases, and the adequacy of the computer program, than with the intrinsic quality of the turbulence model. It is certainly not possible to say that any one class of turbulence model has conclusively proved its superiority over the others, even when cost of computation is ignored. It does, however, appear that Reynolds-stress-transport methods are a distinct improvement over eddy-viscosity methods in complex flows, though both are wounded by the unsatisfactory state of the dissipation-transport equation. The treatment of the viscous wall region, which is not closely linked to the model type, also influences the results.

Perhaps the most telling result was the large range of predictions of flat-plate skin friction - even for different implementations of a single model (the popular "2-equation" (k, ϵ) model based on partial differential equations for turbulent energy and dissipation rate): indeed this range was as large as the differences between independent models. Even after considerable pressure from the organizers, low-speed flat-plate skin-friction predictions still fill a 7 percent band (ignoring outliers with serious discrepancies): taking the wetted area of a civil transport aircraft as five times the wing area, this corresponds to 35 drag "counts", or 35 passengers in a large aircraft.) Numerical inaccuracy was only partly to blame: a remarkable cause of discrepancy was the disagreement over the supposedly-universal "law of the wall", discussed in Appendix 1.

The prediction of compressible flow was a primary interest of the funding agencies: most flat-plate results closely followed the Van Driest correlation of experimental data for the ratio of compressible to incompressible skin friction, which is still believed to be the most reliable - it allows for the effects of mean density variations but ignores compressibility effects (density/pressure fluctuations) as such. (Note that presenting results as the ratio of compressible to incompressible skin friction suppresses the scatter in incompressible c_f discussed above.) The only well-documented flow that shows large Mach-number effects is the mixing layer: "predictions" of the decrease in spreading rate with increasing Mach number either used an *ad hoc* compressibility correction or gave poor results.

The decision to run this project via interaction by mail, rather than as a conference like the 1968 and 1980-81 Stanford meetings, was taken so that participants would have time to consider their early results, compare them with those of others and make minor improvements in their prediction methods. Although this happened, very usefully (in some cases, program bugs of embarrassingly long standing were uncovered) it seems that only a "drop dead" conference deadline can concentrate the minds of the turbulence modeling community enough to produce results on demand. Each section of the community - universities, government establishments, consulting companies - has its own difficulties over manpower, facilities and finance.

Detailed technical results will be discussed in the following sections: administratively, the

main conclusion is that in spite of the rather large rate at which papers on turbulence modeling are being published, some with quite detailed comparisons with experimental data, few groups can rise to the challenge of producing comparisons with *independently-chosen* test data within a reasonable time frame. A number of modelers quoted lack of resources or budget constraints as a reason for lack of response, but we have not heard that any sponsors have refused explicit requests for diversion of funds from the turbulence modeling efforts they support. The organizers have - entirely unofficially - repeatedly pointed out to modelers that the funding agencies, and not necessarily only those supporting the present project, will use the outcome as a guide to how much support to give turbulence modeling in the next few years, and to which groups that support should be directed. Quantity and quality of response has by no means been proportional to the size of the group. U.S. government laboratories have contributed practically no results for the final group of test cases, while two one-man consulting companies were among the most competent and helpful collaborators. Undoubtedly a number of modelers have dropped out simply because they were not able to predict the test cases to an accuracy which they wished to demonstrate in public, but we have no way of distinguishing these from modelers who dropped out through lack of facilities or simply through lack of motivation. Most collaborators were from the United States: on a percentage basis, the enthusiasm and competence of response was no better, and no worse, from the United States than from other countries. It was one of the overseas modelers who pointed out that the research climate has become very much less favorable since the time of the 1980-81 Stanford meeting, for which a large number of groups produced results for a larger number of test cases than those employed in the present collaborative effort. Although no spectacular advances have been made in turbulence modeling in the last decade, it remains a lively subject, with at least 100 high-quality papers being published each year, apparently as the results of basic research rather than deadline-driven development work. It is difficult to see that the modeling community's poor response can be attributed simply to lack of funding.

Details of which modelers attempted which test cases are given in Appendix 4.

2. HISTORY

(The history of the project has been recorded in the five project Newsletters and their attachments, already distributed to the funding agencies. The following is an outline: the sixth Newsletter is an Appendix to this report, and vice versa.)

After discussions at the "Whither Turbulence" meeting at Cornell University in March 1989 (Ref. 3) a proposal for international collaboration on testing of turbulence models was submitted to U.S. Air Force Office of Scientific Research, acting as coordinator for U.S. Army Research Office, NASA and Office of Naval Research. Invitations to participate were sent out in late August 1989, to all originators of turbulence models known to the organizers, to a number of consulting companies and other organizations likely to have well-developed versions of models originated by others, and to all experimenters identified as likely to be able to contribute data or comments.

2.1 "Entry" test cases (flat-plate boundary layers)

In order to calibrate both the models and the modelers (specifically, the time of response of the latter), simple "entry test cases" were distributed in February 1990. The requirement was to predict the skin friction in a turbulent boundary layer in zero pressure gradient at a Reynolds number, based on momentum thickness, of 10,000, in as many as possible of the following cases: (i) low-speed flow, (ii) a Mach number of 5 on an adiabatic wall, (iii) low-speed flow with an absolute wall temperature 6 times the free stream temperature, corresponding approximately to the temperature ratio across the $M = 5$ adiabatic boundary layer. Stanton number (heat transfer) predictions were requested for cases 1 and 3. This set of test cases also had the organizational purpose of identifying modelers who could produce results for compressible flow.

The low-speed high-temperature test case was chosen so that the relative importance of density changes, and of compressibility (Mach number) effects as such, in the various models could be clarified. Many flat-plate skin friction *formulas* (as distinct from detailed prediction methods), use explicit Mach number factors and would not necessarily do well for a low-speed hot wall. In fact most models performed as well for the low-speed hot wall as for the $M = 5$ adiabatic wall, indicating that the models allowed adequately for density changes: indeed, true compressibility effects are probably small in boundary layers up to $M = 5$. It was of course very satisfactory that this test case turned out to be a non-issue.

The "entry" cases proved to be an invaluable calibration. Few modelers managed to keep to the relatively tight deadline imposed for return of results, even for these almost trivial test cases: moreover, as the results began to come in it became obvious that the range of predictions was rather wide. In many cases predictions were, quite simply, outside the possible bounds of experimental error for these simple cases. (In general the organizers have attempted to keep to the error standards appropriate to the aerospace industry: a discrepancy of 0.0001 in skin-friction coefficient – about 3 percent –is big enough to worry about.) A great deal of time and effort was spent on interactions with individual modelers and requests for further information. Probably, a few modelers had calibrated their methods against pipe or duct flow rather than boundary layers, but the main explanation of the really large errors seems to be that many models intended for complex or compressible flows had simply not been adequately checked in simple low-speed flows. Another cause of error was inconsistency in choice of logarithmic-law constants or their equivalent (see Appendix 1). The final results can be described only as a computational catharsis: many modelers submitted revised results after cleaning up empirics, numerical resolution, and downright programming errors. It should be remarked here that the modelers whose results were questioned by the organizers were uniformly grateful.

2.2 August 1990 test data (Cases 3.1-3.5, 4.1-4.3)

We felt it essential to clean up most of the questions over the "entry case" flat-plate computations before proceeding to the next set of test cases, so that it was not until August 1990 that the first real test cases were sent out. They were intended to cover a wide range of flows, keeping as far as possible to thin shear layers and/or simple geometries. We took it for granted that the turbulence models which were most advanced, or most up-to-date, would probably be imbedded in simplified codes, capable of handling only a

limited range of geometries – perhaps to the point of being restricted to thin shear layers. For this reason, we have concentrated throughout the project on test cases which are geometrically simple but physically general.

The "August 1990" data included some recent experiments and simulations, but also test cases from the Stanford 1980-81 conference on complex turbulent flows and from the AGARDograph compilation of compressible-flow data by Fernholz and Finley (Ref. 4). In the case of free shear layers (plane jets, round jets, and mixing layers) detailed experimental results were not given, and modelers were simply asked to compare predicted growth rates with the consensus of experimental data. The only flows requiring a full Navier-Stokes program were the backward-facing steps of Driver and Seegmiller (Ref. 5). The boundary-layer simulation of Spalart and the duct simulation of Moin, Kim and Moser were included, and modelers were asked to compare the *highest-order quantities* they modeled (e.g. dissipation or triple products).

Because of the ongoing discrepancies in the incompressible-flow results, the compressible flows in the August 1990 package were deliberately restricted to one real test case, a boundary layer in strong adverse pressure gradient, plus a second set of "entry" test cases, namely the prediction of flat-plate skin friction for Mach numbers of 2, 3, 5 and 8 and temperatures down to 0.2 of the adiabatic-wall temperature. The corresponding "data" were simply the predictions of the Van Driest II skin-friction formula, which experts in the field regard as being still an acceptable data correlation.

One group of modelers who were disadvantaged by our concentration on thin shear layer data was those who use (Reynolds-averaged) Navier-Stokes codes, which do not easily accept the boundary-layer simplification of specified velocity at the boundary layer edge. Their polite reproaches were entirely justified: a boundary-layer calculation is a solution to only half the problem.

2.3 Results for August 1990 test cases

The speed of response to these test cases was extremely disappointing, with very few results being returned by the specified deadline. A number of modelers stated that they would be able to produce results, although not within the deadline. Since the object of the collaboration was to avoid the "drop dead" deadline of a conference, we, the organizers, decided to wait until a representative body of results had been returned. Obviously this totally disrupted our plans for handling the data, which envisaged an intensive effort beginning at the deadline and accomodating only a few latecomers. In fact, despite promises, only a very few sets of results were returned after Spring 1991. In August 1991 the assembled results were distributed to all the known collaborators, with a request for comments.

To keep down the amount of material to be redistributed, we specified that modelers should return plots only of key quantities (e.g. in the backward-facing-step flow, simply the surface shear stress and the maximum shear stress at each streamwise position), and although many of those modelers who did respond did not complete the full set of "priority" test cases, the stack of graphs distributed was about 1.5″ thick. Some useful comments

have been received, but it is clear that not too many of the experimenters or modelers who had undertaken to join the project were able to devote serious effort to assessing other people's results. Since this was the main reason for calling the project a *Collaboration*, the poor response by the experimenters, coming on top of delays in the computations, was unfortunate.

The results for the thin shear layers were mixed, with no obvious best model. The Wilcox k, ω and multiscale models gave good and closely similar results: the multiscale model has some of the features of an Algebraic Stress Model – a type which was otherwise used only for a few test cases – and this suggests that the improvement shown by ASM-type models over a good two-equation eddy-viscosity model in 2-D thin shear layers may not be significant. The spread of results from the different versions of the k, ϵ model was comparable with, but not closely correlated with, the spread for the "entry" cases. The results for free shear layers showed the usual round-jet / plane-jet "anomaly": few models can predict both flows without some form of special correction factor. Launder's group has recently shown (Ref. 6) that Navier-Stokes calculations for jets produce significantly different results from parabolic ("boundary layer") calculations, partly because of the effect of longitudinal stress gradients, but also partly because of the large effect of longitudinal diffusion of dissipation rate: the modeled transport equation for dissipation is so highly empirical that there may be no physical explanation, but the discrepancy provides a further opportunity for confusion in testing turbulence models.

The number of different models was too small to build up a pattern in the comparisons of "highest-order" quantities with the simulation data. Because of the low Reynolds number of the simulations this was mainly a check on the wall-layer treatments, and the "low-Reynolds-number" versions of the stress-transport models produced tolerably close agreement with the simulations. The simulations show higher dissipation rate in the viscous wall region than do the experiments, and the models seem to have been tuned for the latter. Both experiments and simulations can suffer from errors due to inadequate spatial resolution, most severe near the wall, but on balance the simulations are likely to be more accurate.

Predictions of the backward-facing step flow were surprisingly scattered (even discounting the differences among the k, ϵ models), and most models considerably overestimated the maximum negative skin friction in the recirculating flow, whether they used wall functions or low-Reynolds-number treatments. The simplest explanation is excessive diffusion of momentum into the recirculating flow. Unfortunately no stress-transport models were integrated for the full length of the test flow (32 step heights – expensive in a Navier-Stokes calculation) so that their potential advantage in representing "history" effects has not been demonstrated in this flow.

Predictions of compressible flat plate skin friction up to $M = 8$ and of heat transfer on cold walls at $M = 5$ were mixed. Results for the adiabatic cases were good, with a few exceptions (thought to be models developed for transonic flow and not previously tested at hypersonic speeds). As in the case of low-speed flow, the results depended strongly on the treatment of the wall region: many models reproduce the mixing-length formula with

a constant turbulent Prandtl number, and this is of course the basis of the Van Driest transformation. Recent work, independent of the present project (Ref. 7), has shown that the k, ϵ model does *not* reproduce the Van Driest transformation because of the presence of density gradients in the diffusion terms: however, many users of k, ϵ models obtained results in fair agreement with the Van Driest "predictions".

Results for cold walls showed a wide spread. Confusion occured when several modelers did not realise that Stanton number has to be based on the adiabatic wall temperature actually predicted by the model, *not* that given by a selected value of recovery factor (if the latter is used, St goes to $\pm\infty$ as the predicted adiabatic wall temperature is approached).

Only a few modelers reported results for the compressible boundary layer in strong pressure gradient near $M = 3$, and these results were generally satisfactory (this particular flow happens to have almost constant skin friction coefficient and therefore looks uneventful, but it is a reasonably severe medium-Mach-number test case). The user of one of the only "integral" methods presented pointed out, in connection with this case, that a simple model which has been carefully calibrated may out-perform more advanced models on its home ground. This may be the First Law of Turbulence Modeling.

A few modelers argued that their only concern was with compressible flows and they therefore did not wish to bother with incompressible test cases. This seems a shortsighted attitude: obviously if a model is found to be inaccurate in incompressible flow it cannot be relied on in compressible flow. (Modelers with codes that do not run exactly at $M = 0$ were encouraged to run at, say, $M = 0.4$ and $M = 0.3$ and extrapolate to $M = 0$: most compressibility effects vary as M^2 at low M so there is no difficulty of principle here.)

2.4 August 1991 test data (cases 5.1-5.9)

A second set of "real life" test cases was sent out at the same time as the results for the August 1990 set. These test cases were chosen to explore various complex-flow effects, such as reverse transition, streamline curvature, 3-dimensionality and unsteadiness. Again, most of the test cases were thin shear layers; the 3-dimensional flows in fact had only two independent variables; and the time-dependent flow was homogeneous in the horizontal plane (and therefore computable by trivial adaptation of a 2-dimensional space-marching program).

Two cases were specifically intended to test treatments of the viscous wall region. The first, a simulation of sink-flow boundary layers, gives a simple performance index: does the model predict reverse transition at the same value of pressure-gradient parameter as the simulation? The second, a sinusoidally-oscillating time-dependent flow, in principle causes grief to a "wall function" which uses the friction velocity $\sqrt{(\tau_w/\rho)}$.

Unfortunately, no modeler did the parametric check we requested for the sink flow in sufficient detail to bracket the critical pressure-gradient parameter. To our surprise, two modelers successfully predicted the oscillating flow with wall functions (undoubtedly using $\sqrt{|\tau_w/\rho|}$): presumably their finite time steps did not land them too close to the phase angle at which τ_w changes sign. Amazingly, only one modeler reported results for the curved

boundary layers, although the curved jet flows generated the best response.

Despite our attempts to maintain geometrical simplicity and avoid excluding modelers without curvilinear Navier-Stokes codes, the response to the second set of test cases has been extremely disappointing. Many modelers have stated an inability to devote more effort to the project: undoubtedly some prompt responders have become impatient of the delays caused by the slow responders. A final deadline of 31 May 1992 was imposed, but a few later results were accepted for good reason – and were still arriving in December 1992! The outcome is that the only complex flow for which a worthwhile number of predictions has been received is the backward-facing step flow, which is the complex flow most likely to be used for testing a model during development and is therefore not an entirely independent test case.

3. CONCLUSIONS

The Collaboration has not clearly revealed a "best model": in particular, stress-transport models have not demonstrated a large superiority over eddy-viscosity-transport ("two-equation") models although other reviews have come to the conclusion that they do perform consistently better. The main reason for this lack of clear conclusions is a lack of response from the modelers, particularly the developers of stress-transport models.

To minimize numerical difficulties and the time taken to prepare input data, and to avoid excluding models which were implemented only in simple codes, the organizers tried to ensure that the test cases had simple geometries; as far as possible, the cases were thin shear layers accessible to parabolic "marching" programs. Nevertheless, the performance of any given prediction method, and the fraction of the test cases for which results were reported, seemed to depend much less on the model than on the state of development of the code and the care devoted to checks of grid independence and other numerical issues.

The Collaboration raised a number of general questions about turbulence modeling as well as queries about numerical or other shortcomings of particular methods. It has resulted in correction of errors in several models as well as acting as a clearing-house for facts and opinions.

On balance, the Collaboration has been a success, although it would have been much more fruitful if more modelers had been able to devote effort to producing results for more of the 25 test cases, and if more of the "experimenters" had made a serious effort to return comments on the results.

The reader may have detected a tone of irritation in this report. It should therefore be recorded that modelers' replies to the organizers' stream of requests and reminders were always courteous: on the one occasion when a tardy modeler wrote claiming that "the results are in the mail" – they *were*! The project has indeed done a lot to bring the modeling community together, to point out discrepancies in generally-satisfactory models and, perhaps, to establish the principle that a basic requirement of a model of turbulence (or anything else) is good performance in test cases chosen by someone other than the modeler.

Future plans

This Final Report discharges contractual obligations, but will be considerably augmented later as an unpublished but publicly-available report: in addition, a journal paper will be prepared as an "executive summary" and an advertisement for the report. The hierarchy of available material will then be:-

Journal paper

Final Report (augmented)

Newsletters (including graphs)

Data disks and documentation (used and unused data).

Stanford University will need to make a handling charge for this material, but it is hoped that one or more of the data banks now being planned will also archive the disks.

The 1980-81 Stanford meeting on Computation of Complex Turbulent Flows was hampered by lack of time for consideration of the results presented at the meeting. The present "mail order" effort was hampered by the failure of modelers to keep to the deadlines. On the assumption that there is an ongoing need for public comparisons of turbulence models, a possible compromise for the mid-1990s would be to have two meetings, with an interval of six months or a year: at the first, initial comparisons with test cases would be presented and briefly discussed (as at the 1980-81 meeting); modelers would then reconsider their results and present updated versions for final discussion at the (longer) second meeting. The volume of simulation data is already quite large and an increasing range of complex flows is being covered, so that simulations (direct or large-eddy) would probably provide the largest part of the data sets in a mid-nineties project.

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APPENDIX 1

THE "FLAT PLATE" BOUNDARY LAYER TEST CASES

Flat plate boundary layers, that is, those in zero longitudinal pressure gradient, are one of the most basic test cases for turbulent flows (the others being the 2-dimensional duct or "channel", which is an idealization for which reliable experimental data are scarce, and the circular pipe, for which computations are complicated by the axisymmetric geometry). Because the law of the wall extends from the surface to $y \approx 0.15\delta$, where the velocity has typically risen to 70 percent of the free-stream value, model predictions of flat plate boundary layers are dominated by the assumptions made about the law of the wall. Virtually all turbulence models are compatible with law-of-the-wall scaling, and can thus reproduce a logarithmic velocity profile. This applies whether the region between the surface and the start of the logarithmic law ($y^+ \approx 30$) is predicted by integrating a "low Reynolds number" version of the model down to the wall, or imposed as a "wall function" boundary condition at $y^+ = 30$ (say). A data analysis done by Prof. Donald Coles of the California Institute of Technology for the 1968 Stanford meeting (Ref. 1) recommended $K = 0.41$ and $C = 5.0$ in the standard form of the logarithmic law, and we are not aware of any later review which has specifically challenged Coles' conclusions. A rather wide range of values for K and C is quoted in textbooks, but this scatter is attributable mainly to the difficulty of finding, separately, the slope and intercept of a line which is defined over only a relatively short range (of log y^+): most of the published values of K and C lead to very nearly the same value of U^+ at, say, $y^+ = 100$, which is somewhere near the middle of the logarithmic range in a typical laboratory boundary layer.

When the scattered predictions of flat plate skin friction from the modelers started to come in, we requested them to supply details of their treatment of the universal law of the wall, including their predictions (or assumptions) for U^+ at $y^+ = 100$, hereafter referred to as I_{100}. Figure 1 shows the values of skin friction (modelers' names not identified) plotted against the reported value of I_{100}. Now if a turbulence model as used in the outer part of the boundary layer is left fixed, but the value of I_{100} is changed by changing the wall treatment, then $U_e^+ - I_{100}$ will remain very nearly fixed, so that $U_e^+ \equiv \sqrt{c_f/2}$ changes. The line on Figure 1 was produced by Rodi and Scheuerer at our request, using their standard k, ϵ model with different values of C. If the outer-layer predictions of all the models shown in Figure 1 were identical, the results would lie along the curved line, or at least on at line parallel to it. In fact, there seems to be very little correlation between the values of c_f and the values of I_{100}, in spite of the fact that the version of the plot shown in Figure 1 is the latest, after considerable exhortation of the modelers to clean up their assumptions or predictions of the logarithmic law. Specific requests to modelers to justify using logarithmic law constants other than those recommended by Coles have not produced a very satisfactory response. If Figure 1 is divided into four regions by the Rodi-Scheuerer line and a vertical line $I_{100} = 16.24$, predictions in the first and third quadrants would be made *worse* by imposing the Coles values for the log-law constants, while predictions in the second and fourth quadrants would be made *better*.

It is, of course, impossible to compare outer-layer models logically in the face of these

differing assumptions/results for the law of the wall. Many modelers used the popular k, ϵ turbulence model. When the collaboration began we were warned that there would be no point in having a large number of modelers all using the same model, but in fact comparisons of their results have proved instructive, if not heartening. Several k, ϵ users have modified the empirical constants in the model, but those whose results in Figure 1 are marked with a tail used the "standard" coefficients with - obviously - different wall treatments. The failure of the tailed symbols to lie on a line *parallel* to the curved line in Figure 1 can only be contributed to numerical error in the outer layer (grid dependence or programming mistakes). (Note that grid resolution near the wall is not an issue here, since the law-of-the-wall results are being taken for granted.)

Figure 1, therefore, presents a gloomy picture. The common correlation formulas for flat-plate skin friction agree to within about 2 percent at a momentum thickness Reynolds number of 10,000, and it is clear that many models have simply not been optimized for the flat plate boundary layer. Popular values for the law-of-the-wall constants give values of I_{100} that agree to within 3 or 4 percent, at the outside, but many models use or give results well outside this range. Several models with nominally identical assumptions in the outer layer show discrepancies of several percent in skin friction, even in this numerically-simple flow.

APPENDIX 2

THE MODELS

A2.1 The Process of Turbulence Modeling

The object is always to predict the Reynolds (turbulent) stresses: the Reynolds stress in the $x_i x_j$ plane is $-\rho\overline{u_i u_j}$, where the overbar denotes an average (usually a time average). Exact partial-differential "transport" equations for the stresses can be derived from the Navier-Stokes equations, but their right-hand sides include unknown higher-order statistical quantities. The transport equation for a given Reynolds stress contains a source term which is more-or-less proportional to the mean rate of strain in the plane of that stress, which implies that the ratio of the Reynolds stress to the mean rate of strain varies *less* than the Reynolds stress itself and may therefore be easier to correlate empirically: this ratio is of course the "eddy viscosity", which may be different in different planes. The terms in the Reynolds-stress transport equations all have dimensions [velocity^3/length], and almost all models, of whatever order, assume that the turbulence can be described by one velocity scale and one length scale. This is a sweeping assumption, even when one is concerned only with the larger, Reynolds-stress-producing, eddies.

A2.2 Review Of The Models

Except for one "integral" method, all the methods used partial differential equations for the mean velocity. (Integral methods can in principle be derived by applying the Galerkin technique to PDE models: this is not often done in practice, but it shows that integral

methods are not a class by themselves and are not restricted to crude turbulence models.)

Naturally, modelers were asked to use the same model for all test cases, and to repeat the entry test cases if they made any changes to their models. Many of the major modelers have done exactly this and we have no reason to suppose that the results have been significantly confused by unreported changes. Several multi-person groups, and even some individual workers, used entirely different models at different times during the collaboration. The test cases got progressively harder so there was a tendency for simple models to be replaced, or for their users to drop out entirely. The results for simple models (algebraic eddy viscosity or mixing length, and the one integral method) reinforce the conclusion of the 1980-81 meeting that such models, when carefully tuned, are useful in a restricted range of flows.

The "zonal modeling" technique, in which coefficients are altered from flow to flow by logic in the computer program, was explicitly permitted, on condition of full disclosure, but seems not to have been used. We did specify that the same coefficients should be used in the compressible mixing layer and the compressible boundary layer, mainly because a correction to the coefficients which reproduces the observed decrease in spreading rate of a mixing layer also tends to produce an undesired reduction of skin friction in a boundary layer. (Compressibility corrections developed rapidly during the course of the Collaboration and we have not tried to reach a consensus: the present situation is that all the corrections include adjustable constants multiplying quantities of order M^2, and it is difficult to judge the plausibility of the physics.)

Models can be divided into those which assume a direct relation between the Reynolds stresses and the mean velocity field ("eddy-viscosity methods") and those which solve explicit equations for the stresses ("stress-equation methods"). There is a deep hierarchy of eddy-viscosity methods, but they share the feature that if the mean velocity gradients change suddenly, so also do the Reynolds stresses. The exact transport equations for the Reynolds stresses show that a sudden change in mean-velocity gradient merely produces a sudden change in the rate of growth of Reynolds stresses. That is, in reality the mean-velocity gradient occurs as a source term in a differential equation for the stress, rather than as a factor in an eddy viscosity formula.

A2.2.1 Eddy-viscosity models

Since an eddy viscosity is always the ratio of a turbulence quantity (Reynolds stress) to a mean-flow quantity (mean rate of strain), it is obviously determined by a combination of mean-flow scales and turbulence scales and will be well-behaved only when the two sets of scales are proportional (a definition of "local equilibrium" flow). It is clear in practice that relating the eddy viscosity to the turbulence scales gives better results when the mean-flow scales are strongly perturbed, by pressure gradients or otherwise.

The advantage of eddy-viscosity models is that they will usually give smoothly-varying predictions of Reynolds stresses – obviously, like a laminar flow with a smoothly-varying viscosity. Their disadvantage is identical: they will not reproduce the dependence of Reynolds stresses on mean-flow history and the slow response of Reynolds stresses to

sudden changes in mean flow.

Eddy-viscosity models are often classified by the number of partial differential equations used to describe the turbulence:–

• "Zero-equation" or "algebraic" models relate the eddy viscosity to the velocity and length scales of the mean flow (typically free-stream velocity and boundary layer thickness); those used in the present Collaboration included mixing length, eddy viscosity, and one integral method. "Zero-equation" methods can have some success if they conform to the law of the wall (or the skin-friction laws derived from it) and the corresponding density correlation for compressible flow (amounting to an assumption of constant total temperature in an adiabatic-wall boundary layer). They can be expected to perform well in boundary layers in mild pressure gradients – and, paradoxically, in short regions of very strong pressure gradient where the skin friction is determined by the response of the inner layer and the total pressure in the outer layer changes little (so that prediction of those changes by the turbulence model is not critical). The outer-layer model, for example the assumption that mixing length is proportional to shear layer thickness, necessarily relies on the shear-layer thickness being well-defined. Although eddy-viscosity models such as the Baldwin-Lomax model have been used in quite complicated flows, no predictions other than for boundary layers have been submitted to the present Collaboration.

• "Two-equation" or "eddy-viscosity-transport" methods relate the eddy viscosity to the velocity and length scale of the turbulence. Both make the gross assumption that only *one* scalar velocity scale and *one* scalar length scale suffice to fix the eddy viscosity. Current two-equation" models all use the turbulent kinetic energy k as one variable. They can be classified by the variable $k^m\epsilon^n$ used in the second equation to provide a length scale $k^{3/2}/\epsilon \equiv L$ and thus an eddy viscosity proportional to $k^{1/2}L$. The six Reynolds stresses are then given by

$$-\overline{u_iu_j} = c_\mu k^{1/2}L(\partial U_i/\partial x_j + \partial U_j/\partial x_i)$$

which is nominally a *definition* of c_μ as a dimensionless tensor with indices i and j. In reality c_μ is assumed to be a scalar and in all the methods used in the Collaboration it appears to have been taken as a constant (except for "low Reynolds number" modifications in the viscous wall region). The most popular of the "two-equation" eddy-viscosity models is the k,ϵ model ($m = 0$, $n = 1$ in the above classification. Other models in the (m,n) family which have actually been implemented are k,kL ($m = 5/2$, $n = -1$), k,ω ($m = -1$, $n = 1$) and k,τ ($m = 1$, $n = -1$). It is straightforward to convert from one (m,n) pair to another but the diffusion term in the first model converts to a diffusion term *plus* a *source/sink term* in the second, because the diffusivity is a function of the dependent variables. Since the models are usually formulated without a source/sink term of this sort, the implication is that there are real differences between the different (m,n) combinations, with the further implication that there must be a best (and worst) choice: m and n do not have to be integers, though the physics may become obscure if they are not. This point was not addressed during the Collaboration but deserves future consideration.

• "One-equation" models either use a partial differential equation for a velocity scale and

relate the length scale to the shear-layer thickness, or use a single PDE for eddy viscosity. In the latter case the necessary length scale comes, in effect, from the ratio of the velocity scale to a typical mean velocity gradient. One-equation models (with a PDE for turbulent energy k) were sometimes used in the wall region where a two-equation model was used in the outer layer. This is primarily a numerical simplification, but comparison of results in the Collaboration suggested that performance of two-equation methods in boundary layers in adverse pressure gradient can be improved by using the one-equation model as far out as possible! (The limit is set by the need to match the models, and corresponds, in principle, to the onset of significant transport terms in the length-scale equation.) The recent one-equation models of Baldwin & Barth and of Spalart & Allmaras were not represented in the Collaboration.

A2.2.2 Stress-equation models

These are (usually) term-by-term models of the exact Reynolds-stress transport equations. The object is to avoid using an eddy viscosity for the Reynolds stresses, but gradient-diffusion assumptions are commonly used for the turbulent transport ("diffusion") terms. The Reynolds stresses themselves can yield velocity scales – k, being a scalar, is the natural choice for most purposes – but a length (or time) scale is also needed: most main-stream models use the dissipation, obtained from essentially the same equation as in the k,ϵ model. The key parts of the model are the dissipation-transport equation, and the pressure-strain "redistribution" terms in the stress-transport equations themselves. Nearly all the current models are recognizable descendants of the Launder-Reece-Rodi model of 1975, in turn a generalization of the Hanjalić-Launder thin-shear-layer model of 1972. The main improvements in stress-equation models since the 1980-81 meeting are the enforcement of "realizability" (no physically-impossible negative values, or correlation coefficients outside the range ± 1) and of correct behavior in the two-component limit (e.g. at a solid surface, where v goes to zero faster than u and w). Most of the work has gone into improved modeling of the pressure-strain term, with comparatively little attention to the dissipation equation. There is considerable current interest in the transport equation for ω, a.k.a. ϵ/k, as an alternative to the transport equation for ϵ as such, both in two-equation models and in transport-equation models: this is partly a result of the good performance of Wilcox's models in the current collaboration.

"Algebraic stress models" (ASM), of which one example was used for a few test cases, are stress-transport models with severe simplifications of the mean and turbulent transport terms, resulting in an eddy viscosity model with different values of eddy viscosity for the different Reynolds stresses. A specific advantage over standard two-equation models is that the ASM will at least qualitatively predict stress-induced secondary flow in non-circular ducts. Wilcox's multiscale model falls in the transport-equation family but has some features of the Algebraic Stress Model.

A2.3 NUMERICAL ERRORS

Early in the Collaboration, a paper by J.H. Ferziger was circulated, offering simple advice for testing numerical resolution. Although some of the collaborators themselves urged us

to impose any explicit numerical checks, we decided against it, believing that the more serious errors would be spotted by the collaborators themselves when comparing their results with those of others. In particular, we hoped that the flat-plate test cases would be sufficient to identify serious failures of grid independence close to a solid surface, probably the most critical area in most turbulent flows. Apart from this the only test case which grid independence is likely to have been a serious issue is the flow over backward-facing step, where the singularity in geometry at the top face of the step requires step lengths in the x- and y-directions which are considerably smaller than one wall unit. Again, some modelers submitted revised results after private querying of their initial computations. The curved jets (test cases 5.4 and 5.5) are also likely to cause difficulty in numerics, because of the large angle between the velocity vector and the axis, so that rectangular meshes could lead to large false diffusion.

Appendix 3 Test Case summary and tabulated results

(Tabulations are for simple cases where results could be expressed by a single figure, e.g. skin friction coefficient or growth rate.)

TEST CASES - summary

Entry test cases: flat plate skin friction and heat transfer at momentum-thickness Reynolds number of 10000:

(i) M=0 adiabatic (ii) M=0 heat transfer with small temperature difference (iii) M=0 heat transfer with Tw/Te=6 (iv) M=5 adiabatic.

August 1990 test cases (canonical flows and backstep)

(a) Incompressible flows

3.1 Boundary layer in increasing adverse pressure gradient - 80/81 case 0141, Samuel and Joubert
3.2.1 Boundary layer simulation - Spalart
3.2.2 Duct simulation - Kim, Moin and Moser
3.3.1 Homogeneous turbulence, unstrained - 80/81 case 0371
3.3.2 Homogeneous turbulence, strained - Le Penven et al.
3.3.1 Homogeneous turbulence, sheared - Tavoularis & Karnik
3.4.1 Round jet in still air)
3.4.2 Plane jet in still air)Data correlations
3.4.3 Plane mixing layer in still air)
3.5.1 Backward-facing step in duct with 6 deg. expansion angle - Driver and Seegmiller
3.5.2 Backward-facing step in duct with parallel top wall

(b) Compressible flows

4.1.1 Skin friction on an adiabatic wall at M = 2, 3 and 8
4.1.2 Skin friction and heat transfer at M=5, for T_w/T_{aw} = 0.2, 0.4, 0.6 and 0.8.
4.2 Compressible mixing layer (plot spreading rate against M_c)
4.3.1 Boundary layer of Fernando and Smits.

August 1991 test cases - incompressible complex flows

5.1 Two-dimensional sink-flow boundary layer simulation - Spalart
5.2 Two-dimensional boundary layer in and downstream of a convex bend (stabilizing curvature) - Alving
5.3 Two-dimensional boundary layer in a concave bend (destabilizing curvature) - Johnson
5.4 Two-dimensional stably-curved mixing layer - Castro.
5.5 Normally-impinging jet from a circular nozzle - Cooper
5.6 Single-stream swirling jet in still air - Morse
5.7 "Infinite" 35 deg. swept "wing" - Van den Berg
5.8 Pseudo-Ekman 3D boundary layer simulation - Coleman
5.9 One-dimensional time-periodic boundary layer - Sumer

"Entry Cases" - flat plate test results, momentum-thickness Reynolds number 10000: REVISED 10 June 1992.

Case (i) M=0 adiabatic (ii) M=0 heat transfer with small temperature difference (iii) M=0 heat transfer with Tw/Te=6 (iv) M=5 adiabatic.

"Standard answers", using Van Driest II for compressibility effects and St=1.16cf/2:- (i) cf=0.0026 (ii) St=0.00151, (iii) cf=0.00111, St=0.00064 (iv) cf=0.00094, St=0.

Results in alphabetic order of authors:-

Aupoix/Cousteix: modified k, eps.
(i) cf=0.00269 (ii) St=0.00157 (iii) cf=0.00121, St=0.00073
(iv) cf=0.00100

Bose: alg. mixing length (Cebeci-Smith)
(i) cf=0.00245 (ii) St=0.00143 (iii) cf=0.00115, St=0.000709
(iv) cf=0.00092

Childs: k, eps.
(i) cf=0.00261

Davidson: (a) k, eps. (Chien)
(i) 0.00249
(b) two-layer (Chen-Patel)
(i) 0.00249
(c) ASM
(i) 0.00231

De Bruin/Van den Berg: mixing length
(i) cf=0.00260
(iv)cf=0.00093

Demuren: (a) stress-transport
(i) cf=0.00264
(b) high-Re k, eps.
(i) cf=0.00264 (ii) St=0.00147
(c) low-Re k, eps.
(i) cf=0.00258 (ii) St=0.00150

Ertesvag: stress-transport
(i) cf=0.00230

Friedrich/Haidinger: k, omega
(i) cf=0.00265
(iv) cf=0.00093
Gao/Chow: modified k, eps.
(i) cf=0.00266

Gatski: k, tau
(i) cf=0.00250 (ii) St=0.00140 (iii) cf=0.00110, St=0.00065
(iv) cf=0.00092

Goldberg: hybrid k-L/one-eq.
(i) cf=0.00262 (ii) ---------- (iii) cf=0.00106, St=0.00069
(iv) cf=0.00103

Goulas:
(a) k, eps.
(i) 0.00267

(b) N-H (Nagano) k, eps
(i) 0.00264

Haroutunian/Sun: k, eps.
(i) cf=0.00260 (ii) St=.00157

Hanjalic: stress-transport
(i) cf=0.00264

Humphreys: Cebeci-Smith eddy vis.
(i)cf=0.00260

Kasagi: k, eps. (lo Re)
(i) cf=0.00270

Koh: mod. k, eps.
(i) 0.00260

Lakshminarayana: k, eps. (lo Re)
(i) cf=0.00274 (ii) St=.000168 (iii) cf=0.00137, St=0.00070
(iv)cf=0.00124

S.W. Kim: multi-time-scale
(i) cf=0.00227 (ii) St=0.00157 (iii) cf=0.00101, St=0.00065
(iv) cf=0.00091

Kral: (a) Baldwin-Lomax
(i) cf=0.00248 (ii) - (iii) cf=0.00094, St=0.00048
(iv) cf=0.00094
(b) low-Re k, eps.
(i) to come (ii) - (iii) to come
(iv) cf=0.00113

Majumdar, Govindarajan, Mamachandran: k, eps.
(i) cf= 0.0028 (ii) St=0.00164

Malin/Younis: (a) k, eps model
(i) cf=0.00263 (ii) St=0.0014
(b) stress-transport model
(i) cf=0.00260 (ii) St=0.00152

Marvin group: see separate list, below

Meecham/Rosenblatt: stress-transport
(i) cf=0.00274

Mueller: (a) integral method
(i) cf=0.00234
(iv) cf=0.00089
(b) field method
(i) cf=0.00266 (ii) St=0.00156
(iv) cf=0.00103

Nagano: improved k,eps and 2-eq. heat transfer model
(i) cf=0.00266 (ii) St=0.00159

Parameswaran: k, eps.
(i) cf=0.00280 (ii) St=0.00161

Paveliev: mod. k, eps.
(i) cf=0.00255 (ii) cf=0.00161 (iii) cf=0.00120, St=0.00075
(iv) cf=0.00101

V.C. Patel: 2-layer k, eps.
(i) cf=0.00270

Piquet: Baldwin-Lomax
(i) cf=0.00264
(iv) cf=0.00096

Rodi/Zhu/Scheuerer (a) k, eps. with wall functions
(i) cf=0.00264 (ii) St=0.00147 (iii) cf=0.00104, St=0.00066
(iv) cf=0.000913
Rodi/Zhu/Scheuerer (b) 2-layer
(i) cf=0.00276

Savill: (a) low-Re stress-transport model
(i) cf=0.00263
(b) high-Re stress-transport model
(i) cf=0.00271
(c) high-Re ASM
(i) cf=0.00269

Secundov: one-eq. eddy vis.
(i) cf=0.00240 (ii) St=0.00143 (iii) cf=0.00112, St=0.00070
(iv) cf=0.000802

Shima: stress-transport model
(i) cf=0.00266

Singhal/Avva: k, eps.
(i) cf=0.00261 (ii) St=0.00153 (iii) cf=0.00104, St=0.00061
(iv) cf=0.00106, St=0.00063

B.R. Smith: k, kl
(i) cf=0.00264 (ii) St=0.00156 (iii) cf=0.00109, St=0.00065
(iv) cf=0.000902

P.D. Smith: -
(i) cf=0.00259

So: two-equation (a.k.a. k, eps.)
(i) (external flow calc.)
cf=0.00270
(internal flow calc.)
cf=0.00265

Taulbee: -
(i) cf=0.00265 (ii) St=0.00151 (iii) cf=0.00110, St=0.00069

Tulapurkara:
(a) k, eps.
(i) cf=0.00293, 0.00267 (for constants c_eps_1 and c_eps_2 equal to
(1.44, 1.92) and (1.57, 2.00) respectively.

(b) Cebeci-Smith
(i) cf=0.00254

Walker:alg. eddy vis. plus wall function
(i) cf=0.00274 (ii) St=0.00159 (iii) cf=0.00067, St=0.00039
(iv) cf=0.00073

Wilcox: (a) multiscale model
(i) cf=0.00266 (ii) St=0.00159 (iii) cf=0.00105, St=0.00065
(iv) cf=0.00091
(b) k-omega model (low Re)
(i) cf=0.00270 (ii) St=0.00159 (iii) cf=0.00108, St=0.00065
(iv) cf=0.00094

Yamamoto (all with Karman const. = 0.41 and wall fn.)

(a) k, eps.
(i) cf=0.00266

(b) ASM
(i) cf=0.00244

(c) stress-transport
(i) cf=0.00254

NASA RFE (Marvin) group

Code	Model	Cf x 1000	St x 1000 (Based on Trec, with r = 0.88)
(i) & (ii) (Mach no. = 0.3)			
HuC	k-e (Jones-Launder with Sharma's consts)	2.867	1.704
HuC	q-w (Coakley)	2.696	1.604

HuC	Full Reynolds Stress (Launder-Shima)	2.658	1.545
(iii)	(Mach no. = 0.3)		
HuC	k-e	0.877	0.524
HuC	q-w	1.103	0.647
HuC	Full Reynolds Stress	0.968	0.552
(iv)	(Mach no.= 5)		
		Cf x 1000	Recovery Factor
VVHR	Full Reynolds stress (FRAME model)	0.922	0.881
Ho	k-e (J-L model, V-R Wall function)	0.926	0.789
C	q-w	0.877	0.887
C	k-w (Wilcox)	0.898	0.882
C	k-e (Jones-Launder with Sharma's consts)	0.864	0.879
C	Cebeci-Smith	0.881	0.883
C	Johnson-King	0.830	0.883
HuC	Full Reynolds Stress (Launder-Shima)	0.851	0.841

Cases 3.4.1 - 3.4.3: incompressible jets and mixing layers
Revised 11 June 1992

Thickness used to define spreading rate for jets is distance from centre line to half-velocity point. All flows are in "still air". All calculations were to be done with the same model and constants for each flow.

Name	3.4.1 round jet	3.4.2 plane jet	Ratio round:plane
Childs	-	0.112	-
Chung model 1	0.094	0.100	0.94
.. model 2 (Pope)	0.086	-	0.86
Hanjalic	0.095	0.120	0.79
Launder	0.102	0.108	0.94
.. , scalar profile	(0.136)	(0.131)	(1.04)
Launder Basic RSM	0.110	0.108	1.02
Launder Cubic P-Strain	0.098	0.114	0.86
Malin/Younis	0.111	0.102	1.08
Rodi/Scheuerer (k, eps.)	0.117	0.105	1.11
Secundov	0.099	0.168	0.59
Wilcox (k-omega)	0.096	0.110	0.87
.. (multiscale)	0.091	0.104	0.88
Experiment:			
Panchapakesan and Lumley	0.095	-	
Data correlations:			
(i) S.F. Birch, Boeing	0.086-0.090	0.105	0.84
(ii) Launder, endorsed by W.K. George, Cornell	0.093	0.110	0.85

Notes: Childs' calculations, using a modified k, epsilon model in a compressible-flow code, were done in a restricted streamwise region to reduce computing time. The plane-jet result is believed to be acceptable, but the round-jet result, previously circulated, should be disregarded.
Secundov's comments are attached: since his procedure is carried out automatically by the computer, it counts as a single model - though for present purposes it helps to have details of flow- or geometry-dependent coefficients.

3.4.3 mixing layer

Name	spreading rate (between U/U_e = 0.316 and 0.949)
Childs	0.115
Chung	0.118
Malin/Younis	0.107
Rodi/Scheuerer (k, eps.)	0.0976
Secundov	0.125
Smith (model 1)	0.0913
.. (model 2)	0.117
Wilcox (k, omega)	0.116
.. (multiscale)	0.114
Birch (data correlation)	0.115

We omitted to specify the definition of thickness to be used: the above is the "Birch thickness". "Pitot thickness" between U/U_e = 0.223 and 0.975 has been taken as 1.33 times Birch thickness - both are defined in terms of square of velocity - and the 0.1-to-0.9 thickness used by Malin and Younis has been taken as 1.35 times Birch thickness.

Results are discussed in the main set of Organizers' comments at the start of this Attachment.

Case 3.5.1: backstep with 6 deg. wall
and 3.5.2: backstep with 0 deg. wall
Revised 11 June 1992

Below, x_r is distance to reattachment as multiple of step height; c_f is maximum negative skin-friction coefficient in separated-flow region (based on free-stream velocity upstream of step); and $-uv32_{max}$ is maximum shear stress at x/h=32.

3.5.1 (6 deg)

Name	x_r	$-1000c_f$	$-uv32_{max}$
Expt	8.1	.79	2.74
Goldberg	8.95	1.86	2.17
Goulas std	5.6	0.7	3.4
Goulas mod	4.8	.66	2.9
Leschziner:-			
LRE-WLRE-W	6.33	1.55	3.5?
LRE-W+YC	7.55	1.2	3.5?
LRE+NR	6.4	1.4	3.5?
RSM-EV	7.0	.65	2.8
RSM-DH	5.6	.65	3.1
KEM	5.3	.75	3.6
Majumdar	5.25	.87	3.2
Marvin k,w	8.5	.94	3.2
Para.	5.5	.91	2.5
Rodi 2 layer	6.8	1.14	3.5
Rodi k, eps	5.5	.66	-
Singhal	5.8	.41	3.1

3.5.2 (0 deg)

Name	x_r	$-c_f$	$uv32_{max}$
Expt	6.2	1.05	2.0
Chow	?	?	?
Goldberg	-	-	-
Goulas std	5.0	.78	2.1
Goulas mod	5.6	.78	2.0
Leschziner:- uv32 near-illegible			
LRE-WLRE-W	6.0	1.75	2.0
LRE-W+YC	5.3	1.4	2.0
LRE+NR	5.3	1.6	2.0
RSM-EV	5.3	1.4	2.0
RSM-DH	4.7	1.7	2.0
KEM	4.7	1.6	2.0
Majumdar	4.5	.95	2.1
Marvin k,w	6.5	1.17	1.8
Rodi 2 layer	5.7	1.25	2.5
Rodi k, eps	5.2	.72	-
Singhal	4.8	.50	2.1

Case 4.1.1

Adiabatic wall $c_f/c_{f,\mathrm{Van\ Driest}}$ at $Re_{theta}=10^4$

Name	M=2	M=3	M=5	M=8
Aupoix/Cousteix	1.051	1.062	1.092	1.132
Bose:	1.000	1.007	0.996	1.024
Friedrich	1.068	1.121	1.020	-
Gatski:	0.994	0.964	0.921	0.886
Goldberg:				
(mixed k-L/one eq.)	1.036	1.072	1.103	1.147
Hanjalic	0.944	0.897	0.938	-
Marvin:				
Cebeci-Smith	0.963	0.960	0.964	0.978
k, eps. (wall fn.)	0.995	1.007	1.015	1.137
k, eps. (Chien)	1.006	0.985	0.968	0.980
k, eps. (Sharma)	0.985	0.942	0.891	0.863
k-omega (Wilcox)	0.992	0.987	0.998	0.988
q-omega (Coakley)	0.983	0.972	0.966	0.954
Re stress (FRAME)	-	1.030	1.010	1.010
Re Stress (Shima)	0.964	0.933	0.905	0.851
Muller:				
(integral method)	0.903	0.926	0.976	0.982
(low-Re k, eps.)	0.949	0.966	1.030	1.150
Piquet	1.110	0.971	1.032	-
Rodi/Scheuerer	1.021	1.020	1.011	0.988
Secundov	0.906	0.893	0.879	0.863
Avva/Singhal	1.070	1.110	1.160	1.420
B.R. Smith	1.000	1.000	0.990	0.990
		Sarkar mod. gives 1.04 at M=8		
Wilcox				
(k-omega)	1.031	1.020	0.990	0.911
(multiscale)	1.026	1.013	0.998	0.917

Case 4.1.2

Cold wall $c_f/c_{f,aw}$ and 2 St/c_f at M_e = 5

Name	T_w/T_{aw} =0.2	0.4	0.6	0.8	1.0
Aupoix/Cousteix	1.464	1.316	1.191	1.087	1.000
	1.204	1.208	1.211	1.213	-
Bose	1.029	1.071	1.071	1.033	1.000
	1.079	1.076	1.041	0.925	-
Gatski	1.679	1.475	1.308	1.178	1.000
	1.182	1.183	1.184	1.179	-
Goldberg (k-L/one-eq.)	1.650	1.420	1.260	1.120	1.000
	1.290	1.300	1.310	1.310	-
Marvin Cebeci-Smith	1.916	1.483	1.242	1.079	1.000
	1.188	1.192	1.196	1.188	-
k, eps. (wall fn.)	1.321	1.171	1.087	1.042	1.000
	1.220	1.300	1.410	1.460	-
k, eps. (Chien)	1.341	1.243	-	1.047	1.000
	1.184	1.184	-	1.164	-
k, eps. (Sharma)	1.682	1.326	1.106	1.006	1.000
	1.184	1.188	1.180	1.160	-
k-omega (Wilcox)	1.732	1.421	1.237	1.095	1.000
	1.184	1.188	1.184	1.168	-
q-omega (Coakley)	1.413	1.277	1.143	1.054	1.000
	1.188	1.192	1.196	1.196	-
Re stress (FRAME)	-	1.282	1.165	1.080	1.000
	-	1.370	1.410	1.460	-
Re Stress (Shima)	1.675	1.332	1.136	1.005	1.000
	1.158	1.150	1.121	1.036	-
Rodi/Scheuerer	1.594	1.388	1.215	1.090	1.000
	1.305	1.317	1.339	1.403	-
Secundov	1.397	1.284	1.168	1.072	1.000
	1.211	1.217	1.210	1.181	-
Singhal/Avva	2.287	1.761	1.435	1.135	1.000

	0.049	0.461	1.287	3.762	-
B. Smith	1.520	1.350	1.210	1.090	1.000
	1.180	1.170	1.160	1.110	-
Wilcox					
(k-omega)	1.530	1.340	1.200	1.090	1.000
	1.190	1.200	1.210	1.220	-
(multiscale)	1.470	1.310	1.190	1.080	1.000
	1.230	1.230	1.230	1.240	-

Appendix 4 Modellers' Responses

Asterisks indicate cases reported, and type of model used is in parentheses below modeller's name. "=" before name indicates flat-plate "Entry" results not received or believed out-of-date, "$" means model description form not received, "@" means log-law constants not received.

(a) Incompressible flow

Name \ Case	3.1	3.2.1	3.2.2	3.3.1	3.3.2	3.3.3	3.4.1	3.4.2	3.4.3	3.5.1	3.5.2
Aupoix/Cousteix (mixing length)	*										
@Bose (Alg. eddy vis.)											
@Childs (k,eps.)								*	*		
#Chung (k,eps, gamma)							*	*	*		
De Bruin/v.d.Berg (mixing length)	*	*									
Demuren (low-Re k,eps.)	*		*								
$Gatski (stress)					*	*					
Gao/Chow (mod. k,eps.)	*										
Goldberg (one-eq. k,l)	*									*	
Goulas (k,eps.)	*									*	*
Hanjalic (stress)	*	*	*	*	*	*	*				
$@Kasagi (k,eps.)	*	*	*	*		*					
#@Kawamura (k,eps.)			*								
@S.W. Kim (multi timescale)										*	*
@#$Launder/Craft (RSM)		*					*	*	*		
#?Leschziner (RSM)										*	*
Majumdar ()										*	*
Marvin (k,eps., stress k,omega)	*									*	*
Mueller (k,eps., integral)	*	*									

Incompressible flow (continued)

Name	Case 3.1	3.2.1	3.2.2	3.3.1	3.3.2	3.3.3	3.4.1	3.4.2	3.4.3	3.5.1	3.5.2
Nagano (low-Re k, eps.)	*	*									
Parameswaran (k, eps.)										*	
Patel (2-layer k, eps.)	*	*									
Piquet (mod. k, eps.)	*	*	*								
Rodi (k, eps.)	*	*					*		*	*	*
Savill (ASM)	*										
Secundov (one-eq.)	*	*				*	*		*		
Shima (stress)	*	*	*								
Singhal (k, eps.)	*	*	*							*	*
Smith (k, kl)									*		
Tulapurkara (k, eps.)	*	*	*								
Wilcox (k, omega, Multiscale)	*	*	*	*	*	*	*	*	*		
Younis (k, eps: stress)	*						*		*		
TOTAL	21	14	9	3	3	5	7	4	8	9	7

(b) Compressible flow

Case / Name	4.1.1	4.1.2	4.2	4.3.1
Aupoix/Cousteix	*	*		
Bose	*	*		
Childs			*	
Friedrich	*			
Gatski	*	*	*	
Goldberg	*	*	*	*
Goulas				
Hanjalic	*			
Kasagi				
Marvin	*	*	*	
Mueller	*			*
Patel				
Piquet	*			
Rodi/Scheuerer	*	*		
Savill				
Secundov	*	*	*	
Shima				
Singhal	*	*	*	
B. Smith	*	*		
Tulapurkara				
Wilcox	*	*	*	*
TOTAL	9	7	4	2

(c) Complex flows

Case 5. Name	1	2	3	4	5	6	7	8	9
Bose (Alg. eddy vis.)									
Childs (k,eps.)									
#Chung (k,eps, gamma)									
De Bruin/v.d. Berg (mixing length)									
Gatski (stress)									
Goldberg (one-eq. k,l)	*			*	*				
Goulas (k,eps.)									
Hanjalic (stress)	*				*				*
Kasagi (k,eps.)									
#Kawamura (k,eps.)									
S.W. Kim (multi timescale)									
#Launder/Craft (RSM)				*	*				
#Launder/Graham (k,omega)					*				
#Leschziner (RSM)									
Marvin (k,eps., stress k,omega)									
Mueller (k,eps., integral)							*	*	*
#Nieuwstadt (LES, k,eps.)								*	
Patel (2-layer k,eps.)									
Rodi (k,eps.)					*	*			*
Savill (ASM)									
Shima (stress)	*	*	*						
Singhal (k,eps.)									
Smith (k, kl)									

Complex flows (continued)

Case 5.	1	2	3	4	5	6	7	8	9
Name									
Tulapurkara (k,eps.)									
Wilcox (k, omega, Multiscale)									
TOTAL	3	1	1	2	5	1	1	2	3

(d) Summary for test-case groups 3, 4, 5

("cf" and "I100" are skin friction coefficient at momentum thickness Reynolds number 10000, and value of U^+ at $Y^+=100$

	cf	I100	3 1	3 2 1	3 2 2	3 3 1	3 3 2	3 3 3	3 4 1	3 4 2	3 4 3	3 5 1	3 5 2	4 1 1	4 1 2	4 2	4 3 1	5 1	5 2	5 3	5 4	5 5	5 6	5 7	5 8	5 9
Aupoix/Cousteix (mod. k,eps.)	269	16.34												*	*											
@Bose (Alg. eddy vis.)	245	16.70												*	*											
@Childs (k,eps.)	261	16.73							*	*	*															
#Chung (k,eps, gamma)	?								*	*	*															
De Bruin/v.d.Berg (mixing length)	260		*	*																						
$Gatski (k,tau)	250	16.25												*	*											
Goldberg (one-eq. k,l)	262	17.20												*	*			*			*	*				
Goulas (k,eps.)	267																									
Hanjalic (stress)	264													*				*				*				*
$@Kasagi (k,eps.)	270																									
#@Kawamura (k,eps.)	?																									
@S.W. Kim (multi timescale)	227																									
@#$Launder/Craft/ Graham(RSM)	?																				*	*				
#?Leschziner (RSM)	?																									
Marvin (k,eps., stress k,omega, C-S)														*	*											

Mueller (k,eps., integral)	266		**			***
Nieuwstadt (k, eps; LES						*
Patel (2-layer k, eps.)						
Piquet B-L	266		*			
Rodi (k,eps.)			**		**	*
Savill (ASM)						
Secundov			**			
Shima (stress)				***		
Singhal (k,eps.)			**			
B.R. Smith (k,kl)			**			
Tulapurkara (k,eps.)						
Wilcox (k,omega)	269	15.92	**			
Wilcox (Multiscale)	266	16.06	**			

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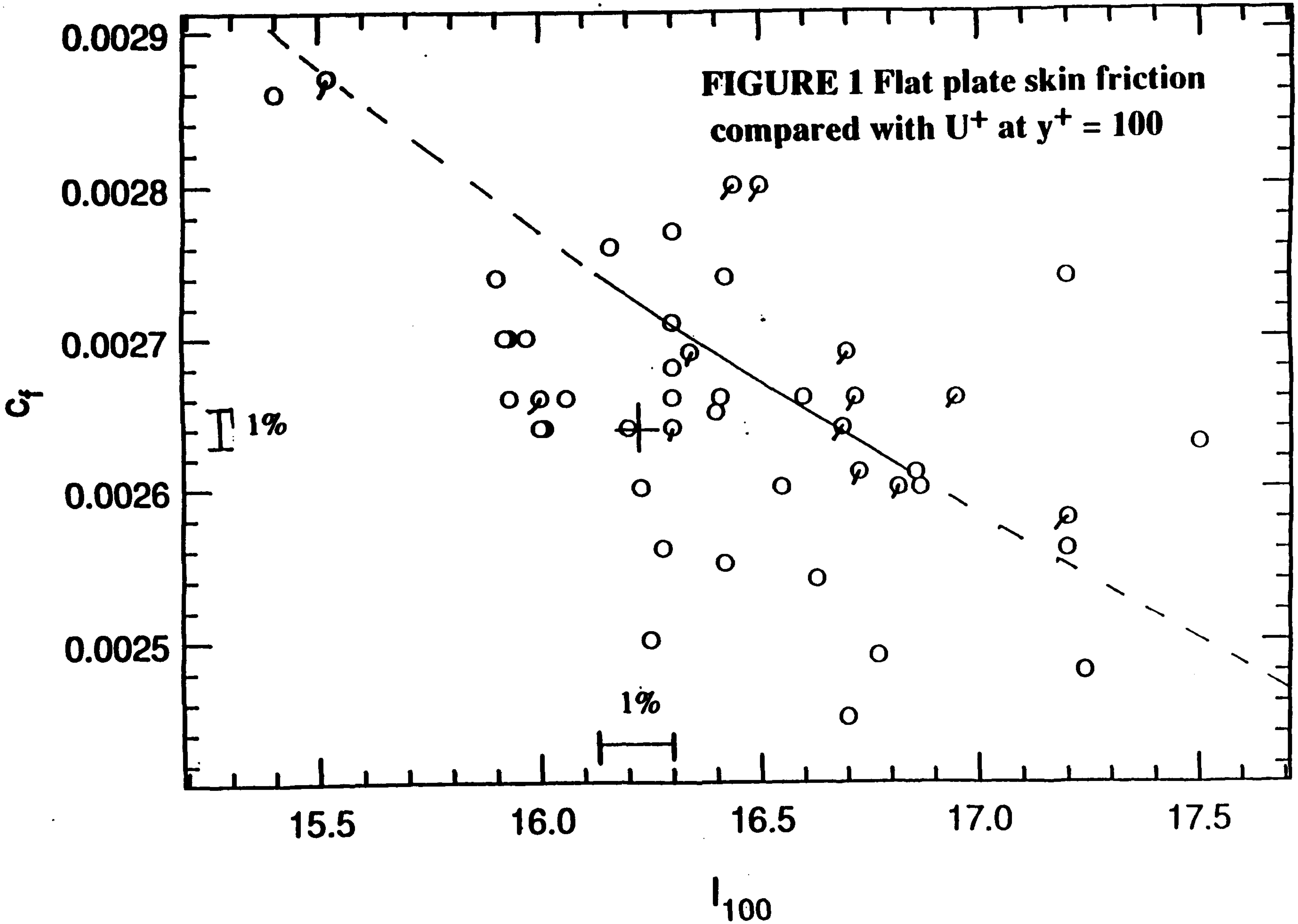

FIGURE 1 Flat plate skin friction compared with U^+ at $y^+ = 100$